NHZ frequency hopping sequence sets under aperiodic Hamming correlation: tighter bound and optimal construction

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Abstract

In this paper, we first establish a new bound on no-hit-zone (NHZ) frequency hopping (FH) sequence sets under aperiodic Hamming correlation. The new bound is tighter than the bound on NHZ FH sequence sets under aperiodic Hamming correlation which was derived by Liu et al. in 2018. Further, we construct a class of NHZ FH sequence sets under aperiodic Hamming correlation. They are optimal with respect to the new bound and have more flexible parameters than those in [13].

1 Introduction

In frequency hopping (FH) communication systems, FH sequences play an important role. In such systems, each user is represented by a sequence of hopping frequencies.

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Simultaneous transmission by any two users over the same frequency band results in collisions of signals, and hence, it is very desirable that such collisions over the same frequency band are minimised. The degree of such collisions is clearly related to the Hamming correlation properties of the FH sequences. The Hamming correlation functions are very important parameters of FH sequences. It is expected to design FH sequences with small Hamming correlation values [2, 3, 4, 6, 12]. However, Hamming correlation values are bounded by some other parameters of FH sequences such as sequence length, family size, frequency slot set size, and so on [17, 18]. Then FH sequences cannot be designed with small Hamming correlation values that we expect.

In quasi-synchronous FH communication systems, the time delay is limited in a certain range, namely low hit zone/no hit zone (LHZ/NHZ) [19]. In LHZ/NHZ, FH sequences can be designed with low Hamming correlation value [22]. If the time delay is controlled within LHZ/NHZ, then interferences can be reduced effectively. In recent years, some LHZ FH sequence sets are reported in the literature [1, 5, 9, 11, 14, 15, 16, 20, 21]. For NHZ FH sequences, as long as the time delay is within NHZ, the Hamming correlation value of them is equal to zero.

The Hamming correlation functions generally fall into two categories: periodic Hamming correlation and aperiodic Hamming correlation. Aperiodic Hamming correlation can measure FH communication system performances more accurately than periodic Hamming correlation. Nevertheless, the aperiodic Hamming correlation of FH sequences was rarely studied in the literature. Although there are some results on FH sequence sets under aperiodic Hamming correlation in the literature [7, 8, 10], they are not NHZ FH sequence sets. In 2018, Liu et al. [13] first established a bound on NHZ FH sequence sets under aperiodic Hamming correlation and gave a construction for them, which are optimal with respect to the bound. There is no other result on NHZ FH sequence sets under aperiodic Hamming correlation in the literature.

In this paper, we study NHZ FH sequence sets under aperiodic Hamming correlation. First, we derive a new bound on NHZ FH sequence sets under aperiodic Hamming correlation, which is tighter than the bound obtained by Liu et al. [13]. Moreover, we give a construction of NHZ FH sequence sets under aperiodic Hamming correlation. The new sequence sets are optimal with respect to the new bound and have more flexible parameters than those in [13].

The rest of this paper is organized as follows. In Section 2, the related definitions and notations are introduced. In Section 3, a new bound on NHZ FH sequence sets under aperiodic Hamming correlation is derived. In Section 4, a construction of NHZ FH sequence sets under aperiodic Hamming correlation is presented. Finally, the paper concluded with some remarks in Section 5.

2 Definitions and Notations

Let $V = \{l_0, l_1, \cdots, l_{v-1}\}$ be a frequency slot set with size v and $F = \{F^0, F^1, \cdots, F^{K-1}\}$ a set of FH sequences over V, where $F^i = (f_0^i, f_1^i, \cdots, f_{L-1}^i)$ for $i = 0, 1, \cdots, K-1$. Let $c(x, y) = 1$ for $x = y$ and $c(x, y) = 0$ for $x \neq y$. The aperiodic Hamming correlation of F^i

and F^j at time delay t is given by

$$C_{F^iF^j}(t) = \sum_{k=0}^{L-t-1} c(f^i_k, f^j_{k+t}). \tag{1}$$

For the FH sequence set F, the maximum aperiodic Hamming correlation $C(F)$ is defined by

$$C(F) = \max\{C_a(F), C_c(F)\}, \tag{2}$$

where

$$C_a(F) = \max\{C_{F^iF^i}(t)|F^i \in F,\ t = 1, 2, \cdots, L-1\}$$

and

$$C_c(F)=\max\{C_{F^iF^j}(t)|F^i, F^j \in F,\ i \neq j,\ t=0, 1, \cdots, L-1\}.$$

There are some results on FH sequence sets under aperiodic Hamming correlation in the literature [7, 8, 10]. However, they are not NHZ FH sequence sets under aperiodic Hamming correlation.

Let F be an NHZ FH sequence set under aperiodic Hamming correlation. The NHZ N of F is defined by

$$N = \min\{N_a, N_c\}, \tag{3}$$

where

$$N_a = \max\{n|C_{F^iF^i}(t) = 0, F^i \in F, t = 1, 2, \cdots, n\}$$

and

$$N_c = \max\{n|C_{F^iF^j}(t)=0, F^i, F^j \in F, i \neq j, t=0, 1, \cdots, n\}.$$

Definition 1: Let $[L, K, v, N]$ denote an NHZ FH sequence set under aperiodic Hamming correlation over a frequency slot set with size v, whose sequence length, family size, NHZ are L, K, N, respectively.

In 2018, Liu et al. [13] first obtained the following bound on NHZ FH sequence sets under aperiodic Hamming correlation.

Theorem 1: For an $[L, K, v, N]$ NHZ FH sequence set under aperiodic Hamming correlation, we have

$$\left\lceil \frac{L}{N+1} \right\rceil \geq \frac{LK}{v}. \tag{4}$$

3 New Bound on NHZ FH Sequence Sets under Aperiodic Hamming Correlation

In this section, we derive a new bound on NHZ FH sequence sets under aperiodic Hamming correlation which is tighter than the bound (4).

Theorem 2: For an $[L, K, v, N]$ NHZ FH sequence set under aperiodic Hamming correlation, we have

$$N \leq \frac{v}{K} - 1. \tag{5}$$

Proof: Let $F = \{F^0, F^1, \cdots, F^{K-1}\}$ be an $[L, K, v, N]$ NHZ FH sequence set under aperiodic Hamming correlation, where $F^i = (f_0^i, f_1^i, \cdots, f_{L-1}^i)$ for $i = 0, 1, \cdots, K-1$. By the definition of NHZ, for any $0 \leq i, j \leq K-1$, $0 \leq t \leq N$, $(i-j)^2 + t^2 \neq 0$, we have

$$C_{F^i F^j}(t) = 0. \tag{6}$$

By (1) and (6), we have

$$c(f_k^i, f_{k+t}^j) = 0 \tag{7}$$

for $k = 0, 1, \cdots, L-t-1$. This implies that in the set $\{f_k^i | i = 0, 1, \cdots, K-1,\ k = 0, 1, \cdots, N\}$ whose size is $K(N+1)$, the elements are different from each other. Since the size of the frequency slot set is v, we have

$$K(N+1) \leq v. \tag{8}$$

This leads to

$$N \leq \frac{v}{K} - 1$$

as desired.

□

Definition 2: If NHZ N is the maximum integer solution of (4) or (5), then the corresponding NHZ FH sequence set under aperiodic Hamming correlation is optimal with respect to the corresponding bound.

The new bound (5) is tighter than the bound (4) because of

$$\frac{L}{N+1} \leq \left\lceil \frac{L}{N+1} \right\rceil.$$

We illustrate this by Figure 1. It can be seen that for sequence length $L = 24$ and family size $K = 2$ the new bound (5) is always tighter than or equal to the bound (4) as frequency slot set size v changes.

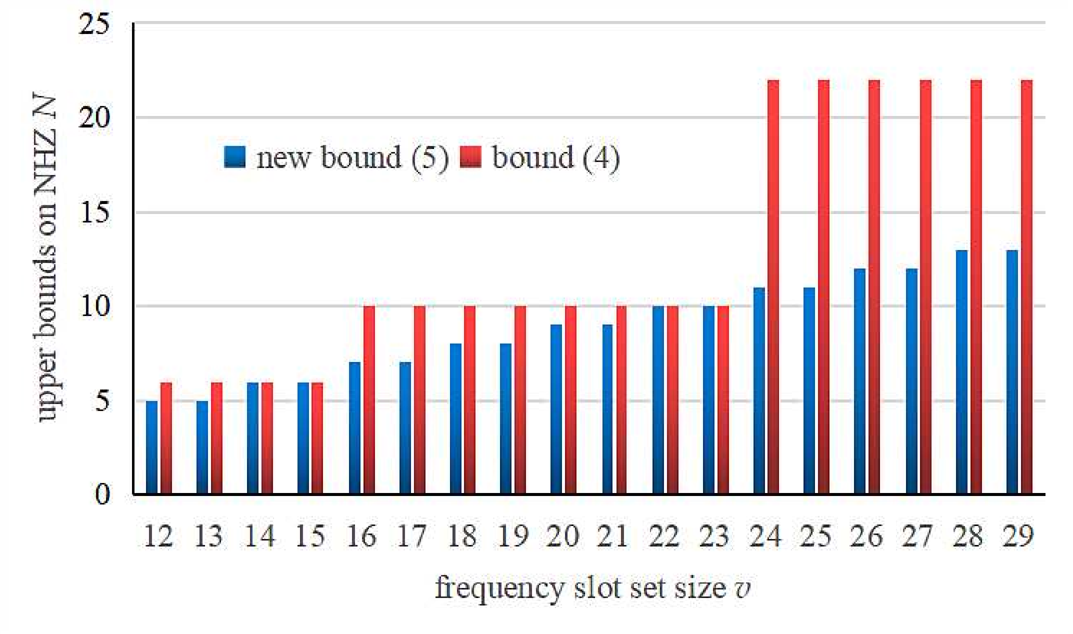

Figure 1: Upper bounds on NHZ N for sequence length $L = 24$ and family size $K = 2$

4 Optimal Construction for NHZ FH Sequence Sets under Aperiodic Hamming Correlation

In this section, we construct a class of NHZ FH sequence sets under aperiodic Hamming correlation which are optimal with respect to the new bound (5).

Construction:

Step 1. Let l, k be two integers. For $V = \{0, 1, \cdots, v' - 1\}$, we choose a base matrix B over V as follows

$$B = \begin{pmatrix} b_0 & b_k & \cdots & b_{\lfloor \frac{v'}{k} \rfloor k-k} \\ b_1 & b_{k+1} & \cdots & b_{\lfloor \frac{v'}{k} \rfloor k-k+1} \\ \vdots & \vdots & & \vdots \\ b_{k-1} & b_{2k-1} & \cdots & b_{\lfloor \frac{v'}{k} \rfloor k-1} \end{pmatrix},$$

where $b_0, b_1, \cdots, b_{\lfloor \frac{v'}{k} \rfloor k-1}$ are distinct.

Step 2. Construct a $k \times l$ matrix M as follows

$$M = \begin{pmatrix} b_0 & b_k & \cdots & b_{\lfloor \frac{v'}{k} \rfloor k-k} & b_{\lfloor \frac{v'}{k} \rfloor k} & \cdots & b_{lk-k} \\ b_1 & b_{k+1} & \cdots & b_{\lfloor \frac{v'}{k} \rfloor k-k+1} & b_{\lfloor \frac{v'}{k} \rfloor k+1} & \cdots & b_{lk-k+1} \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ b_{k-1} & b_{2k-1} & \cdots & b_{\lfloor \frac{v'}{k} \rfloor k-1} & b_{\lfloor \frac{v'}{k} \rfloor k+k-1} & \cdots & b_{lk-1} \end{pmatrix},$$

where for any $i = \left\lfloor \frac{v'}{k} \right\rfloor, \left\lfloor \frac{v'}{k} \right\rfloor + 1, \cdots, l-1$,

$$b_{ik}, b_{ik+1}, \cdots, b_{ik+k-1} \in V \backslash \{b_{ik-\lfloor \frac{v'}{k} \rfloor k+k}, b_{ik-\lfloor \frac{v'}{k} \rfloor k+k+1}, \cdots, b_{ik-1}\} \tag{9}$$

and are distinct.

Step 3. Construct an FH sequence set $F' = \{F^0, F^1, \cdots, F^{k-1}\}$ where

$$F^i = (b_i, b_{k+i}, b_{2k+i}, \cdots, b_{lk-k+i}),\ i = 0, 1, \cdots, k-1. \tag{10}$$

For the FH sequence set F', we have the following theorem.

Theorem 3: F' is an $[l, k, v', \left\lfloor \frac{v'}{k} \right\rfloor - 1]$ NHZ FH sequence set under aperiodic Hamming correlation and optimal with respect to the new bound (5).

Proof: For the FH sequence set F', the aperiodic Hamming correlation of F^i and F^j at time delay t is given by

$$C_{F^iF^j}(t) = \sum_{m=0}^{l-t-1} c(b_{mk+i}, b_{(m+t)k+j}). \tag{11}$$

Case 1. $i = j$ and $0 < t \le \left\lfloor \frac{v'}{k} \right\rfloor - 1$. Note that $(m+t)k+j-(mk+i) = tk \le \left\lfloor \frac{v'}{k} \right\rfloor k - k$. If $(m+t)k+j \le \left\lfloor \frac{v'}{k} \right\rfloor k - 1$, then it is obvious that $b_{mk+i} \ne b_{(m+t)k+j}$. We consider the case $(m+t)k+j > \left\lfloor \frac{v'}{k} \right\rfloor k - 1$. By (9), we have

$$b_{(m+t)k+j} \in V \backslash \{b_{(m+t)k-\lfloor \frac{v'}{k} \rfloor k+k}, b_{(m+t)k-\lfloor \frac{v'}{k} \rfloor k+k+1}, \cdots, b_{(m+t)k-1}\}.$$

Since $(m+t)k - \left\lfloor \frac{v'}{k} \right\rfloor k + k \le mk+i \le (m+t)k-1$, we have

$$b_{mk+i} \in \{b_{(m+t)k-\lfloor \frac{v'}{k} \rfloor k+k}, b_{(m+t)k-\lfloor \frac{v'}{k} \rfloor k+k+1}, \cdots, b_{(m+t)k-1}\}.$$

This implies that $b_{mk+i} \ne b_{(m+t)k+j}$. Hence $C_{F^iF^j}(t) = 0$.

Case 2. $i \ne j$ and $0 \le t \le \left\lfloor \frac{v'}{k} \right\rfloor - 1$. It is obvious that $mk+i \ne (m+t)k+j$. If $t = 0$, then $b_{mk+i} \ne b_{(m+t)k+j}$. If $t \ne 0$, then $mk+i \le (m+t)k+j$. Similar to Case 1, we can get that $b_{mk+i} \ne b_{(m+t)k+j}$. Thus $C_{F^iF^j}(t) = 0$.

Therefore F' is an $[l, k, v', \left\lfloor \frac{v'}{k} \right\rfloor - 1]$ NHZ FH sequence set under aperiodic Hamming correlation.

For an $[l, k, v', N]$ NHZ FH sequence set under aperiodic Hamming correlation, by bound (5) we have

$$N \le \left\lfloor \frac{v'}{k} - 1 \right\rfloor = \left\lfloor \frac{v'}{k} \right\rfloor - 1.$$

Thus, F' is optimal with respect to the new bound (5).

□

Example: Let $l = 13$, $k = 3$, $v' = 10$. A $[13, 3, 10, 2]$ NHZ FH sequence set under aperiodic Hamming correlation $F' = \{F^0, F^1, F^2\}$ can be constructed, where

$$F^0 = (5, 9, 3, 8, 1, 4, 7, 0, 9, 6, 2, 0, 8),$$
$$F^1 = (8, 0, 4, 2, 5, 3, 6, 1, 4, 7, 5, 9, 3),$$
$$F^2 = (2, 1, 6, 7, 0, 9, 2, 5, 8, 3, 1, 4, 6).$$

Table 1. The parameters of NHZ FH sequence sets under aperiodic Hamming correlation in [13] and this paper

Parameters $[L, K, v, N]$	Restriction	According to the bound (4)	According to the new bound (5)	Reference
$[l', k', v'', \lfloor \frac{v''}{k'} \rfloor - 1]$	$\lfloor \frac{v''}{k'} \rfloor + 1 \big\vert l'$	Optimal	Optimal	[13]
$[l, k, v', \lfloor \frac{v'}{k} \rfloor - 1]$		Uncertain	Optimal	This paper

It is easy to verify that the aperiodic Hamming correlation is 0 for NHZ $N = 2$. For the $[13, 3, 10, N]$ NHZ FH sequence set under aperiodic Hamming correlation, by bound (4) we have

$$N \leq 3.$$

By new bound (5) we have

$$N \leq 2.$$

Thus, F' is optimal according to the new bound (5) but not optimal according to the bound (4).

Table 1 lists the parameters of NHZ FH sequence sets under aperiodic Hamming correlation in [13] and this paper. It is easy to see that the FH sequence sets in [13] are restricted by the condition $\lfloor \frac{v''}{k'} \rfloor + 1 \big\vert l'$, while the FH sequence sets in this paper are not restricted by any condition. The parameters of FH sequence sets in this paper are more flexible than those in [13].

5 Conclusions

In this paper, a new bound on NHZ FH sequence sets under aperiodic Hamming correlation was derived which is tighter than the bound derived by Liu et al. in 2018. Moreover, a construction for NHZ FH sequence sets under aperiodic Hamming correlation was given. The new FH sequence sets are optimal with respect to the new bound. They also have flexible parameters compared with those in [13]. It is expected that the new bound can be used for more FH sequence designs and the new FH sequence sets are useful in quasi-synchronous FH communication systems to eliminate interference.

References

[1] J. H. Chung and K. Yang, "New classes of optimal low-hit-zone frequency-hopping sequence sets by Cartesian product," *IEEE Trans. Inf. Theory*, vol. 59, no. 1, pp. 726-732, Jan. 2013.

[2] J. H. Chung and K. Yang, "Optimal frequency-hopping sequences with new parameters," *IEEE Trans. Inf. Theory*, vol. 56, no. 4, pp. 1685-1693, Apr. 2010.

[3] C. Ding and J. Yin, "Sets of optimal frequency-hopping sequences," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3741-3745, Aug. 2008.

[4] P. Z. Fan, M. H. Lee, and D. Y. Peng, "New family of hopping sequences for time/frequency-hopping CDMA systems," *IEEE Trans. Wireless Commun.*, vol. 4, no. 6, pp. 2836-2842, Nov. 2005.

[5] H. Y. Han, D. Y. Peng, and P. Udaya, "New sets of optimal low-hit-zone frequency-hopping sequences based on m-sequences," *Cryptogr. Commun.*, vol. 9, no. 4, pp. 511-522, Jul. 2017.

[6] A. Lempel and H. Greenberger, "Families of sequences with optimal Hamming correlation properties," *IEEE Trans. Inf. Theory*, vol. 20, no. 1, pp. 90-94, Jan. 1974.

[7] X. Liu and D. Y. Peng, "Sets of frequency hopping sequences under aperiodic Hamming correlation: upper bound and optimal constructions," *Advances in Mathematics of Communications*, vol. 8, no. 3, pp. 359-373, Aug. 2014.

[8] X. Liu and D. Y. Peng, "Frequency hopping sequences with optimal aperiodic Hamming correlation by interleaving techniques," *Advances in Mathematics of Communications*, vol. 11, no. 1, pp. 151-159, Feb. 2017.

[9] X. Liu, D. Y. Peng, and H. Y. Han, "Low-hit-zone frequency hopping sequence sets with optimal partial Hamming correlation properties," *Designs, Codes and Cryptography*, vol. 73, no. 1, pp. 167-176, Oct. 2014.

[10] X. Liu and Q. Zeng, "Frequency hopping sequence sets with good aperiodic Hamming correlation properties and large family size," *IEEE Communications Letters*, vol. 23, no. 3, pp. 394-397, Mar. 2019.

[11] X. Liu and L. Zhou, "New bound on partial Hamming correlation of low-hit-zone frequency hopping sequences and optimal constructions," *IEEE Communications Letters*, vol. 22, no. 5, pp. 878-881, May 2018.

[12] X. Liu, L. Zhou, and S. Q. Li, "A new method to construct strictly optimal frequency hopping sequences with new parameters," *IEEE Trans. Inf. Theory*, vol. 65, no. 3, pp. 1828-1844, Mar. 2019.

[13] X. Liu, L. Zhou, and Q. Zeng, "No-hit-zone frequency hopping sequence sets with respect to aperiodic Hamming correlation," *Electronics Letters*, vol. 54, no. 4, pp. 212-213, Feb. 2018.

[14] W. Ma and S. Sun, "New designs of frequency hopping sequences with low hit zone," *Designs, Codes and Cryptography*, vol. 60, no. 2, pp. 145-153, Aug. 2011.

[15] X. H. Niu, D. Y. Peng, and Z. C. Zhou, "New classes of optimal low hit zone frequency hopping sequences with new parameters by interleaving technique," *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, vol. E95-A, no. 11, pp. 1835-1842, Nov. 2012.

[16] X. H. Niu, D. Y. Peng, and Z. C. Zhou, "New classes of optimal frequency hopping sequences with low hit zone," *Advances in Mathematics of Communications*, vol. 7, no. 3, pp. 293-310, Aug. 2013.

[17] D. Y. Peng and P. Z. Fan, "Lower bounds on the Hamming auto- and cross-correlations of frequency-hopping sequences," *IEEE Trans. Inf. Theory*, vol. 50, no. 9, pp. 2149-2154, Sep. 2004.

[18] D. Y. Peng, P. Z. Fan, and M. H. Lee, "Lower bounds on the periodic Hamming correlations of frequency hopping sequences with low hit zone," *Science in China: Series F Information Sciences*, vol. 49, no. 2, pp. 1-11, Jun. 2006.

[19] D. Y. Peng and H. Y. Han, "Frequency/time hopping sequences with no hit zone: bounds and designs," *Journal of Chengdu University of Information Technology*, vol. 30, no. 1, pp. 1-6, Feb. 2015.

[20] L. M. N. Zhou, D. Y. Peng, H. B. Liang, C. Y. Wang, and H. Y. Han, "Generalized methods to construct low-hit-zone frequency-hopping sequence sets and optimal constructions," *Cryptogr. Commun.*, vol. 9, no. 6, pp. 707-728, Nov. 2017.

[21] L. M. N. Zhou, D. Y. Peng, H. B. Liang, C. Y. Wang, and Z. Ma, "Constructions of optimal low-hit-zone frequency hopping sequence sets," *Designs, Codes and Cryptography*, vol. 85, no. 2, pp. 219-232, Nov. 2017.

[22] Z. C. Zhou, X. H. Tang, and G. Gong, "A new class of sequences with zero or low correlation zone based on interleaving technique," *IEEE Trans. Inf. Theory*, vol. 54, no. 9, pp. 4267-4273, Sep. 2008.